Reg. No. : [| | | | | | | | | | |]

Question Paper Code : 80871

B.E/B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

Fourth Semester

Computer and Communication Engineering

MA 8451 — PROBABILITY AND RANDOM PROCESSES

(Common to : Electronics and Communication Engineering/ Electronics and Telecommunication Engineering)

(Regulations 2017)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Two events A and B have following probabilities: $P[A] = 0.5$, $P[B] = 0.6$ and $P[A \cap B] = 0.24$. Find the value of $P[A' \cap B']$.

2. Find moment generating function of geometric distribution.

3. If X_1 has mean 4 and variance 9 while X_2 has mean –2 and variance 5 and the two are independent, find $\operatorname{Var}(2X_1 + X_2 - 5)$.

4. The joint PMF of two random variables X and Y is given by $P_{XY}(x, y) = \begin{cases} k(2x+y), & x = 1,2;\ y = 1,2 \\ 0, & \text{otherwise} \end{cases}$.
Determine the value of the constant k.

5. State the four types of stochastic processes.

6. When is a stochastic process said to be ergodic?

7. Check whether the function $\dfrac{1}{1+9\tau^2}$ is valid auto correlation function.

8. Write power spectral density function of a WSS process.

9. Define average power in the response of a linear system.

10. If the input to a linear time invariant system is white noise $\{N(t)\}$, what is power spectral density function of the output?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Three car brands A, B, and C, have entire market share in a certain city. Brand A has 20% of the market share; brand B has 30% and brand C has 50%. The probability that a brand A car needs a major repair during the first year of purchase is 0.05, the probability that a brand B car needs a major repair during the first year of purchase is 0.10, and the probability that a brand C car needs a major repair during the first year of purchase is 0.15.

(1) What is the probability that a randomly selected car in the city needs a major repair during its first year of purchase? (4)

(2) If a car in the city needs a major repair during its first year of purchase, what is the probability that it is a brand A car? (4)

(ii) Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability for each of the following events:

(1) Exactly two messages arrive within one hour. (3)

(2) No message arrives within one hour. (2)

(3) Atleast three messages arrive within one hour. (3)

Or

(b) (i) A random variable X is uniformly distributed between 3 and 15. Find the following parameters.

(1) The expected value of X. (2)

(2) The variance of X. (2)

(3) The probability that X lies between 5 and 10. (2)

(4) The probability that X is less than 6. (2)

(ii) The savings bank account of a customer showed an average balance of Rs.150 and a standard deviation of Rs.50. Assuming that the account balances are normally distributed.

(1) What percentage of account is over Rs.200? (3)

(2) What percentage of account is between Rs.120 and Rs.170? (3)

(3) What percentage of account is less than Rs.75? (2)

12. (a) Random variables X and Y have joint PDF

$$f_{XY}(X,Y) = \begin{cases} 4xy, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & otherwise \end{cases}$$

(i) What are $E[X]$ and $\text{Var}[X]$? (4)

(ii) What are $E[Y]$ and $\text{Var}[Y]$? (3)

(iii) What is $\text{Cov}[X, Y]$? (3)

(iv) What is $E[X+Y]$? (3)

(v) What is $\text{Var}[X+Y]$? (3)

Or

(b) Marks obtained by 10 students in Mathematics (x) and Statistics (y) are given below :

x	25	28	35	32	31	36	29	38	34	32
y	43	46	49	41	36	32	31	30	33	39

Find

(i) The two regression lines (6)

(ii) Coefficient of correlation between the marks in Mathematics and Statistics (6)

(iii) y when $x = 30$ (4)

13. (a) Define semi random telegraph signal process and random telegraph signal process and also prove that the semi random telegraph signal process is evolutionary and the random telegraph signal process is WSS. (16)

Or

(b) (i) Prove that the difference of two independent Poisson processes is not a Poisson process. (6)

(ii) An Engineer analyzing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals followed a highly distorted signal with no recognizable signal, whereas 20 out of 23 recognizable signals follow recognizable with no highly distorted signals between. Given that only highly distorted signals are not recognizable. Find the fraction of signals that are highly distorted. (10)

14. (a) Let the random process be $W(t) = X(t) \cos \omega t + Y(t) \sin \omega t$ where $X(t)$ and $Y(t)$ are two jointly stationary processes. What are the conditions for $W(t)$ to be a WSS? In case $W(t)$ is WSS, what is its autocorrelation in times of autocorrelations of $X(t)$ and $Y(t)$? (16)

Or

(b) (i) The power spectrum of WSS process $\{X(t)\}$ is given $S_{XX}(\omega) = \dfrac{1}{(1+w^2)^2}$. Find the auto correlation function and hence the average power. (8)

(ii) If the cross-correlation of two process $\{X(t)\}$ and $\{Y(t)\}$ is $R_{XY}(t, t+\tau) = \dfrac{AB}{2}\{\sin(\omega_0 \tau) + \cos(\omega_0(2t+\tau)\}$ where A, B and ω_0 are constants. Find the cross-power spectrum. (8)

15. (a) Let $X(t)$ be the input voltage and $Y(t)$ be the output voltage with system transfer function $H(\omega) = \frac{R}{R + j\omega L}$. Also $\{X(t)\}$ is a stationary process with $E(X) = 0$ and $R_{XX}(\tau) = e^{-\alpha|\tau|}$. Find $E(Y)$, $S_{YY}(\omega)$ and $R_{YY}(\tau)$.

Or

(b) Let $Y(t) = A\cos(\omega_0 t + \theta) + N(t)$ where A is a constant. θ is a random variable with a uniform distribution in $(-\pi, \pi)$ and $\{N(t)\}$ is band limited Gaussian white noise with a power spectral density.

$$S_{NN}(\omega) = \begin{cases} \frac{N_0}{2} & |\omega - \omega_0| \le \omega_B \\ 0, & otherwise \end{cases}.$$

Find the spectral density function of $\{Y(t)\}$ assuming that $N(t)$ and θ are independent.

———————